Schrodinger Equation

$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$

$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \frac{1}{2}m\omega^2x^2\Psi = E\Psi$

$TISE\ -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi$

Op form $H\psi = \left(\frac{1}{2m}p^2 + \frac{1}{2}m\omega^2x^2\right)\psi = E\psi$

Infinite Square Well

$P_{ab} = \int_a^b \rho(x)dx$

$\Psi^*\Psi = probability\ density$

$\langle H\rangle = \int d\tau\Psi^*\hat{H}\psi = E = \sum|c_n|^2E_n$

$\langle H^2\rangle = \int d\tau\Psi^*\hat{H}(\hat{H}\psi) = E^2$

$\sigma_H^2 = \langle H^2\rangle - \langle H\rangle^2$

$\hat{H}\psi_n = E_n\psi_n$

$k = \frac{\sqrt{2mE}}{\hbar}\quad k = \frac{n\pi}{a}$

$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$

$\psi(\vec{r},t) = \sum c_n\psi_n(\vec{r})e^{-\frac{iE_nt}{\hbar}}$

Eigenvalues $E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2 \Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$

$c_n = \int \psi_n(x)^*f(x)dx = \int \frac{\phi_n}{\sqrt{L}}dx$

$= \frac{1}{\sqrt{L}}\sqrt{\frac{2}{L}}\int_0^L \sin\frac{n\pi x}{L}dx$

Harmonic Oscillator

$[x,p] = i\hbar\quad [A,B] \equiv AB - BA$

$Hamiltonian\ H = \frac{\vec{p}^2}{2m} + V(r) \to \hat{H} = \frac{-\hbar}{2m}\nabla^2 + V(r)$

also $H = \left(a_+a_- + \frac{1}{2}\right)\hbar\omega$

$E_n = \left(A_n + \frac{1}{2}\right)\hbar\omega \Rightarrow E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}e^{\frac{-m\omega x^2}{2\hbar}}$ & $\psi_n = \frac{(a_+)^n}{\sqrt{n!}}$

$a\pm = \frac{1}{\sqrt{2m\hbar\omega}}(m\omega x \mp ip)\ where$

$p = -i\hbar\left(\frac{\partial}{\partial x}\right)$

$a^+\psi_n = \sqrt{n+1}\psi_{n+1}\quad a_-\psi_n = \sqrt{n}\psi_{n-1}$

$N\psi_n = a_+a_-\psi_n = n\psi_n$

$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)\quad p = i\sqrt{\frac{\hbar}{2m\omega}}(a_+ - a_-)$

$x^2 = \frac{\hbar}{2m\omega}(a_+a_+ + a_-a_- + a_-a_+ + a_+a_-)$

$\langle v\rangle = \frac{1}{2}m\omega^2\langle x^2\rangle = \frac{1}{2}m\omega^2\int_{-\infty}^{\infty}dx\psi_n^*x^2\psi_n$

$V(x) = \frac{1}{2}kx^2 \to V = -\int Fdx = \frac{1}{2}m\omega^2x^2$

$F = -kx = m\frac{d^2x}{dt^2}$

Recursion formula/Hermite Polynomials

$a_{j+2} = \frac{2j+1-k}{(j+2)(j+1)}a_j \to \frac{2j-2n}{(j+2)(j+1)}a_j$

$\xi = \sqrt{\frac{m\omega}{\hbar}}x\quad Normalized\ \psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)\frac{1}{\sqrt{2^nn!}}H_n(\xi)e^{-\frac{\xi^2}{2}}$

$\langle p\rangle = \psi^*|\hat{p}|\psi\quad \langle x^2\rangle = \psi^*|x^2|\psi\quad k = \frac{2E}{\hbar\omega}$

$\alpha = \sqrt{\frac{m\omega}{\hbar}}$

$a\pm = \alpha x \mp \beta p$ w/ $\alpha = \sqrt{\frac{m\omega}{2\hbar}}$ & $\beta = \frac{1}{\sqrt{2m\hbar\omega}}$

Free Particle

$k = \frac{\sqrt{2mE}}{\hbar} > 0$

$k = \pm\frac{\sqrt{2mE}}{\hbar}\begin{cases}k > 0 \Rightarrow traveling\ to\ right\\ k < 0 \Rightarrow traveling\ to\ left\end{cases}$

$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = E\Psi \to \frac{\partial^2\psi}{\partial x^2} = -k^2\psi\quad v_{ph} = \frac{\hbar k}{2m}$

Eigenstate $\psi(x) = Ae^{ikx} + Be^{-ikx}$

Stationary State $\Psi_k(x,t) = (Ae^{ikx} + Be^{-ikx})e^{\frac{-i\hbar k^2t}{2m}}$

Square Well/Step Potentials

$k = \sqrt{\frac{2m(E-V)}{-\hbar}}\ w/\begin{cases}CA \Rightarrow e^{\pm ikx}\ or\ sin/cos\\ CF \Rightarrow e^{\pm kx} or\ sinh/cosh\end{cases}$

$\begin{matrix}Discontinuity\\ Representation\end{matrix}\begin{cases}e^{ikx} \Rightarrow \to (trans)\\ e^{-ikx} \Rightarrow \leftarrow (refl)\\ e^{kx} \Rightarrow increas\ exp\\ e^{-kx} \Rightarrow decres\ exp\end{cases}$

$E > V\quad T = \frac{4k_1 * k_2}{(k_1 + k_2)^2}$

$T \equiv \left|\frac{J_{trans}}{J_{inc}}\right| = \frac{\frac{4k_2}{k_1}}{\left(1+\frac{k_2}{k_1}\right)^2}\quad \frac{C}{A} = \frac{2}{1+\frac{k_2}{k_1}}$

$R = 1 - T = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}\quad R \equiv \left|\frac{J_{ref}}{J_{inc}}\right|$

$= \left|\frac{1 - \frac{k_2}{k_1}}{1 + \frac{k_2}{k_1}}\right|\quad \frac{B}{A} = \frac{1 + \frac{k_2}{k_1}}{1 - \frac{k_2}{k_1}}$

Gram-Schmidt Orthogonalization

$v_1 = u_1\quad v_2 = u_2 - \frac{\langle u_2|v_1\rangle}{\langle v_1|v_1\rangle}v_1$

$v_n = u_n - \sum_{i=1}^{n-1}\frac{\langle u_n|v_i\rangle}{\langle v_i|v_i\rangle}v_i$

Matrices

Inverse $\equiv T^{-1}T = TT^{-1} = I$

Unitary Matrix $U^{-1} = U^{dagger}$, columns & rows each form orthonormal sets

$Ta = \lambda a \to (T - \lambda I)a = 0 \to \det(T - \lambda I) = 0\ (char\ eq)$

For M, $\det(M - \lambda) = 0$ allows eigenvalues to be found

Plug back into $\det(M - \lambda)$ to find eigenvectors

Trace $\equiv \sum$ diagonal matrix terms (frm top left to bottom right)

Integrals

$\int_0^{\infty} e^{-\alpha x^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$

$\int x\sin^2 ax\,dx = \frac{x^2}{4} - \frac{x\sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}$

Other Equations

Laplacian $\nabla^2f = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)f$

$j = \frac{\hbar}{2mi}(\Psi^*\nabla\Psi - (\nabla\Psi^*)\Psi)\quad p = \hbar k$

Uncertainty Principle $\sigma_x\sigma_p \geq \frac{\hbar}{2}$

$\delta(cx) = \frac{1}{|c|}\delta(x)\quad \int f(x)\delta(x-a)dx = f(a)$

$\int d\tau\psi_m^*\psi_n = \delta_{mn}$

David M Vermillion

PHYS 420-001

Fall 2018