Use of a 1mW Laser to Verify the Speed of Light

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Abstract
The lab was set up to verify the accepted value of the speed of light. A 1mW 650nm laser set to 2.97 $\pm$ 0.01 MHz, oscilloscope, frequency generator, flat mirror, sensor assembly, meter stick, fiberglass measuring tape, and four pairs of unaided human eyes were used. They were set up and calculations were performed in accordance with the methods section. Results were calculated to give a value and error margin of $3.04 * 10^8 \pm 0.188 * 10^8 \frac{m}{s}$. The accepted value of *c* lies within the error margin of this calculated measurement. The accepted value for the speed of light was therefore verified in the lab.

Introduction
The lab was set up to verify the speed of light. Starting with the assumption that the speed of light is a constant verifiable value in vacuum[8] and that air has a refractive index of 1.0003[4], an experiment was set up, executed, and evaluated to find the speed of light to compare with the accepted value of *c*.[8]

Over the course of my academic career, it has been drilled into my brain by numerous professors, graduate students, textbooks, homework problems, and other assorted resources that the speed of light is a constant as measured in vacuum from any inertial reference frame.[7] A method for measuring the speed of light was first developed by Galileo, and the first good measurement was made by Ole Römer in 1676.[5] Measurements steadily improved until we decided in the 1970s to define the speed of light and the meter based off of the second, leading to the currently accepted value.[3]

Methods
For the experiment, a 1mW 650nm laser set to 2.97 $\pm$ 0.01 MHz, oscilloscope, frequency generator, flat mirror, sensor assembly, a meter stick, a fiberglass measuring tape, and four pairs of unaided human eyes were used in accordance with instructions given in the instruction manual.[6] The laser was set up on a movable table next to and in line with the sensor.

A measuring tape droop correction formula[10] $\frac{W^2L}{24P^2}$, where W (measuring tape weight per meter) = 0.32 $\frac{N}{m}$ (estimate), L = measured distance, and P (estimated tape pull) = $\frac{0.5988N}{m}$ was used with regards to the fiberglass measuring tape. Distances with the measuring tape were measured in inches from the laser edge to the mirror reflecting surface edge and the mirror reflecting surface edge to the sensor edge, then converted in Excel using CONVERT(cell,"in","cm"). Four significant figures were entered for use in actual calculations to avoid roundoff error.

A measurement was taken at 1.0 $\pm$ 0.5 cm with a measured value of 146 $\pm$ 2 ns for our zero value. A second measurement was made at 340 $\pm$ 4 cm but discarded because of a lack of independent value verification (only one measurement of distance and time instead of four as with the rest). The remaining measurements are detailed in the following table.

Measurements in Table Form		
Measurement Number	Distance (cm)	Time (ns)
0	1.0 ± 0.5 cm	146 ± 2 ns
1	336 ± 4 cm	170 ± 2 ns
2	453.6 ± 5.4 cm	170 ± 2 ns
3	1176 ± 14 cm	180 ± 2 ns
4	1992 ± 24 cm	208 ± 2 ns

Calculations from the measurements were made in the form of $\frac{d_2-d_1}{t_2-t_1}$, which should theoretically provide 10 calculations from the form $\sum_{i=1}^{5} x-1$. However, because measurements 1 and 2 are so close, $\frac{m2}{m1}$ results in division by zero, and $\frac{m3}{m2}$ and $\frac{m3}{m2}$ result in absurd outliers because they were more than twice the value of the other individually calculated results. Consequently, only 7 comparisons were considered. Of these, $\frac{d_4-d_3}{t_4-t_3}$ is the closest measurement to the accepted value of *c*. Subsequent measurements should be able to make a better value.

Error was calculated using Average Deviation $\frac{\sum_{i=1}^{n}|(x_i-x_{avg})|}{N}$, a measuring tape droop correction formula[10] $\frac{W^2L}{24P^2}$, where W (measuring tape weight per meter) = 0.32 $\frac{N}{m}$ (estimate), L = measured distance, and P (estimated tape pull) = $\frac{0.5988N}{m}$, oscilloscope movements of 2 ns, a measuring tape reading error of 2 cm, and error propagation formulas: [multiplication/division ⇒ $\frac{\Delta D}{D}=\left|\frac{\Delta A}{A}\right|+\left|\frac{\Delta B}{B}\right|$] and [addition/subtraction ⇒ $A\pm\Delta A\pm B\pm\Delta B=C,\ \Delta C=\Delta A+\Delta B$]. Only Average Deviation, reading, and droop error were considered when finding the average distance and average time because error propagation was negligible.

Sample Calculations

Average Deviation for measurement 2 laser flight time in ns:

$$\frac{|(162-168.5)+(166-168.5)+(174-168.5)+(172-168.5)|}{4}=1.625\ ns$$

Measuring tape droop for measurement 1: Correction = $\frac{(1.07N)^2(3.34m)}{24(2N)^2}*\frac{100cm}{1m}=3.98\ cm$

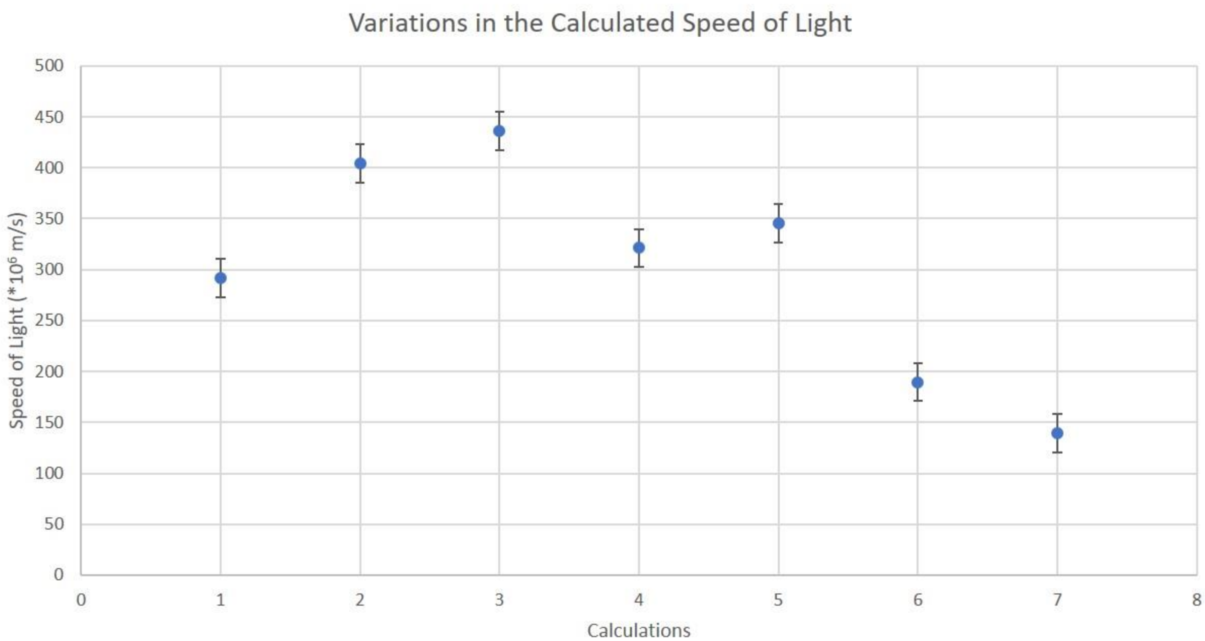
Oscilloscope average time for measurement 1 of 169.5 ns rounds to 170 $\pm$ 2 ns.

Error propagation for the average time of measurement 1 in nanoseconds. $\frac{\Delta E}{E}=\left|\frac{3.5\ ns}{166\ ns}\right|+\left|\frac{-2.5\ ns}{172\ ns}\right|+\left|\frac{-4.5\ ns}{174\ ns}\right|+\left|\frac{3.5\ ns}{166\ ns}\right|=1\ ns$

Inch to centimeter conversion: =CONVERT(130.9,"in","cm") = 332.5 cm

Results

Using the form $\frac{d_2-d_1}{t_2-t_1}$ to evaluate all possible combinations, one was discarded as not mathematically tenable and two were discarded as severe outliers because they were more than twice the value of the other individually calculated results. The remaining numbers to three significant figures resulted in $\boldsymbol{c=3.04*10^8\pm0.188*10^8\frac{m}{s}}$. A scatter-plot follows to illustrate the large variance in calculated values.


Variations in the Calculated Speed of Light
Speed of Light (*10^6 m/s)
500
450
400
350
300
250
200
150
100
50
0
0
1
2
3
4
5
6
7
8
Calculations

Discussion

The lab confirmed the hypothesis that the speed of light is $3.00 * 10^8 \frac{m}{s}$.[8] The result of the experiment resulted in a calculated value of $c = 3.04 * 10^8 \pm 0.188 * 10^8 \frac{m}{s}$.[1] The accepted value of *c* is within the error margin of the lab results. The closest single measurement was $\frac{d_4 - d_3}{t_4 - t_3}$, which gave a result of $2.91 * 10^8 \pm 0.188 * 10^8 \frac{m}{s}$.[1] More measurements should result in a much more accurate measurement, both closer to *c* and with less error.

Measurements 1 and 2 had flight times that are close enough to each other than some large error is assumed. It is known that the frequency shifted to such an extent after being measured at 0 that it reached 3.20 $\pm$ 0.01 MHz on measurement 3 which makes measurements 2 and 3 suspect. Whether the frequency generator shifted on its own, or if we messed something up accidentally while experimenting is unknown. The unanticipated frequency increase likely moved the peaks in ways that were unaccounted for.

As stated in the methods section, because of the faulty measurements in 2 and 3, three combinations were untenable. More measurements would have resulted in both better proficiency with the equipment, reducing error, and more data points, which would have presumably closed the error gap.

The first two hours were spent trying to figure out what to do to make the equipment work correctly. The instruction manual said that 10 to 20 meters of clear space were needed. That one sentence caused an unnecessary number of problems. Based on this experience, we should at least start with the “suggested” method in the equipment manual until after two successful data points have been gathered. Then methods can be tweaked as necessary.

This lab has been very different from those I have been accustomed to doing. It makes the process more challenging, but also more rewarding.

Conclusion

Within a large margin of error, the speed of light was verified in the lab. This verification resulted in a value that within the error margin, is where the speed of light is accepted to reside. Specifically, the experimental result was that $c = 3.04 * 10^8 \pm 0.188 * 10^8 \frac{m}{s}$. This confirms that the speed of light can be easily calculated with minimal equipment and readily verified.

References

1. David Vermillion, Lab 1 Calculations.xlsx, 25 January 2018.
2. Group A Lab, North Carolina Agricultural and Technical State University, Physics Department, Intermediate Laboratory, 23 January 2018.
3. How Do We Measure the Speed of Light?, Forbes, https://www.forbes.com/sites/quora/2016/11/07/how-do-we-measure-the-speed-of-light/#723b972a21d2, 3 February 2018.
4. Maya Barsky, Index of Refraction of Air, 2005. https://hypertextbook.com/facts/2005/MayaBarsky.shtml.
5. Michael Fowler, The Speed of Light, UVA Physics Department, http://galileoandeinstein.physics.virginia.edu/lectures/spedlite.html, 3 February 2018.
6. PASCO Laser Speed of Light Apparatus Model No. AP-8586 Instruction Manual.
7. Special Relativity, http://physics.bu.edu/py106/notes/Relativity.html, 3 February 2018.
8. Speed of Light, The NIST Reference on Constants, Units and Uncertainty, https://physics.nist.gov/cgi-bin/cuu/Value?c|search_for=speed+of+light, 31 January 2018.
9. Standard Acceleration of Gravity, The NIST Reference on Constants, Units and Uncertainty, https://physics.nist.gov/cgi-bin/cuu/Value?gn, 31 January 2018.
10. Tape Corrections, Civil Engineering, http://www.civilengineeringx.com/surveying/tape-corrections/, 31 January 2018.