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The Alcubierre Warp Drive

An Introductory Mathematical Overview and Analysis of Current Standing

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Abstract

Warp Drive as a means of faster-than-light (FTL) travel has only recently begun to be treated academically in a semi-serious manner as a thought experiment, starting with Dr. Miguel Alcubierre's 1994 seminal paper, *The Warp Drive: Hyper-Fast Travel within General Relativity*. The papers referenced in this examination look at Warp Drive deriving from a loophole in the Einstein Field Equations. Warp Drive achieves speeds above the speed of light through creating a bubble of spacetime such that the interior can be stationary (and below the speed of light) while the bubble moves by warping spacetime itself. This paper contributes both a derivation of the Alcubierre warp drive spacetime metric and graphical sampling of a sub-component equation. Some examination of select articles addressing warp drive yields a snapshot of current literature and iterative published thoughts. While at this point, warp drive is purely theoretical, it has future uses in space exploration and colonization. Space exploration on this scale will improve scientific understanding in a huge cross-section of fields on the same level as our existing space exploration has improved our understanding compared to a century ago. Similarly, being able to travel to the extent warp drive allows would improve the odds of the human species surviving a variety of possible existential crises. Further theoretical developments are necessary before anyone could begin effective experimental tests. The current literature is somewhat scattered. Further condensing and analyzing literature could produce more actionable results.

Introduction

Warp Drive as a means of faster-than-light (FTL) travel has been a staple of science fiction lore since at least the debut of Star Trek in 1966 [1]. However, its first semi-serious treatment was by Dr. Miguel Alcubierre in 1994 with his seminal paper, *The Warp Drive:*

Hyper-Fast Travel within General Relativity [2]. Since then about 800 papers have been written by various authors iteratively finding more concrete methods of utilizing theoretical physics and mathematics to improve the theoretical framework of non-locally FTL propulsion. I mention several of these authors, including Natário, Lobo, and Van Den Broeck in my first examination of warp drive [3]. Most systems assume an open loophole in General Relativity to achieve non-locally FTL propulsion. This paper examines the main physical concepts behind Warp Drive as a specific type of FTL travel, provides worked mathematics on one equation left unworked in Alcubierre's original paper, and shows illustrations of a sub-component equation and how it changes with parameter variance. It concludes with a statement about the current state of research and offers suggestions for the next steps in theoretical development.

Background

General Relativity is a mathematical and physical framework developed by Albert Einstein in 1916 [4]. In his breakthrough paper, Einstein expands upon the earlier work of Special Relativity to show that the fabric of spacetime is affected not only by relative speeds through the γ Lorentz Factor defined as $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ where $c \equiv$ the speed of light and $v \equiv$ the speed of an object, but spacetime is also affected by the strength of gravitational fields through the tensor Einstein Field Equation $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$ [5]. This paper briefly explores how parsing some loopholes the Einstein Field Equation represents allows for the theoretical creation of a warp drive.

The foundational work of Alcubierre was to show that working from General Relativity, it is possible to break the universal speed limit c according to an outside observer through the creation of a spacetime bubble. Inside the confines of the spacetime bubble, the traveling spaceship could be stationary, while according to an outside observer, the bubble is traveling any arbitrary speed $\gg c$. Upon arrival to the destination and computing speed through the classical formula $\frac{\Delta distance}{\Delta time}$, the spaceship occupants would find their speed to have also been $\gg c$. This is computed through the spacetime metric $ds^2 = -dt^2 + (dx - v_s f(r_s)dt)^2 + dy^2 + dz^2$, discussed in further depth below.

Deriving the Spacetime Metric

One of the steps made in Alcubierre's paper is to show that the spacetime metric given in tensor form can be rewritten into spacetime form. Given the spacetime metric

$$ds^2 = -d\tau^2 = g_{\alpha\beta}dx^\alpha dx^\beta = -\left(\alpha^2 - \beta_i\beta^i\right)dt^2 + 2\beta_i dx^i dt + \gamma_{ij}dx^i dx^j$$

it can be rewritten as

$$ds^2 = -dt^2 + (dx - v_s f(r_s)dt)^2 + dy^2 + dz^2$$

However, while he makes this statement and the individual terms are defined, he does not show the derivation in the paper. Therefore, I will show the derivation below using highlighting colors to show where each part matches up. First, I will define a few terms before beginning derivation.

$$G = c = 1$$

$$\alpha = 1$$

$$\beta^x = -v_s(t)f\left(r_s(t)\right) \longrightarrow -vf \text{ for calculation simplicity}$$

$$\beta^y = \beta^z = 0$$

$$\gamma_{ij} = \delta_{ij}$$

Derivation

Expanding the targeted end equation, we find five terms

$$ds^2 = -dt^2 + (dx - v_s f(r_s)dt)^2 + dy^2 + dz^2$$

$$= -dt^2 + (dx - vfdt)^2 + dy^2 + dz^2$$

$$= -dt^2 + dx^2 - 2vfdxdt + v^2f^2dt^2 + dy^2 + dz^2$$

$$= (v^2f^2 - 1)dt^2 + dx^2 - 2vfdxdt + dy^2 + dz^2$$

These five terms are matched up after working through the tensor form of the spacetime equation. First the tensors will be expanded, then they will be equated to show what they represent. I define spacetime coordinates in this paper as

$$x^0 = ct = t \quad x^1 = x \quad x^2 = y \quad x^3 = z$$

While expanding the tensor form of the spacetime equation, only 5 of the possible 16 terms will survive. The other 4 are omitted for brevity since they are all equal to zero.

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$$

$$= g_{\alpha 0}dx^\alpha dx^0 + g_{\alpha 1}dx^\alpha dx^1 + g_{\alpha 2}dx^\alpha dx^2 + g_{\alpha 3}dx^\alpha dx^3 + \cdots$$

Only g_{00} , g_{11} , g_{01} , g_{22} , and g_{33} are non-zero terms where

$$g_{\alpha\beta} = g_{00}dx^0dx^0 + g_{01}dx^0dx^1 + g_{11}dx^1dx^1 + g_{22}dx^2dx^2 + g_{33}dx^3dx^3$$

$$= g_{00}dt^2 + g_{01}dtdx + g_{11}dx^2 + g_{22}dy^2 + g_{33}dz^2$$

Setting each side equal, the terms become:

$$g_{00}dt^2 = (v^2f^2 - 1)dt^2 \Rightarrow g_{00} = v^2f^2 - 1$$

$$g_{01}dtdx = -2vfdxdt \Rightarrow g_{01} = -2vf$$

$$g_{11}dx^2 = dx^2 \Rightarrow g_{11} = 1$$

$$g_{22}dy^2 = dy^2 \Rightarrow g_{22} = 1$$

$$g_{33}dz^2 = dz^2 \Rightarrow g_{33} = 1$$

This results in the respective equations looking like this:

$$(v^2f^2 - 1)dt^2 - 2vfdtdx + dx^2 + dy^2 + dz^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\beta_i dx^i dt + \gamma_{ij}dx^i dx^j$$

Recognizing that $\alpha = 1$ and the yellow equation is the same on both sides by a factor of -1 such that $-\left(\alpha^2 - \beta_i \beta^i\right)dt^2 = \left(\beta_i \beta^i - \alpha^2\right)dt^2$. Further recognizing that $\gamma_{ij} = \delta_{ij}$, the γ term becomes the green, purple, and grey equations. Also, only $\beta^x = -vf$ survives for all β terms. $\therefore$

$$-\left(\alpha^2 - \beta_i \beta^i\right)dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

$$= \left(\beta_i \beta^i - \alpha^2\right)dt^2 + 2\beta_i dx^i dt + \delta_{ij} dx^i dx^j$$

$$= ((\beta^x)^2 - 1)dt^2 + 2\beta^x dxdt + dx^2 + dy^2 + dz^2$$

$$= (v^2 f^2 - 1)dt^2 - 2vf dxdt + dx^2 + dy^2 + dz^2$$

$$\therefore$$

$$(v^2 f^2 - 1)dt^2 - 2vf dxdt + dx^2 + dy^2 + dz^2$$

$$= (v^2 f^2 - 1)dt^2 + dx^2 - 2vf dxdt + dy^2 + dz^2$$

$$0 = 0$$

$$Q.E.D.$$

Explaining the Top Hat Function of the Spacetime Metric

The spacetime metric discussed above is the equation describing how the bubble moves along a geodesic. One of the sub-component descriptions caught my eye because Dr. Alcubierre described it as a top hat function. Upon initial inspection, it may appear to be a jumbled mess.

$$f(r_s) = \frac{\tanh\left(\sigma(r_s + R)\right) - \tanh\left(\sigma(r_s - R)\right)}{\tanh\left(\sigma R\right)}$$

However, after plotting the equation and playing around with its parameters, a few properties coalesce. First, at a large distance, with $\sigma = 1$ and $0 < R \geq 1$, the equation almost looks like a Kronecker Delta Function as illustrated in Figure 1.

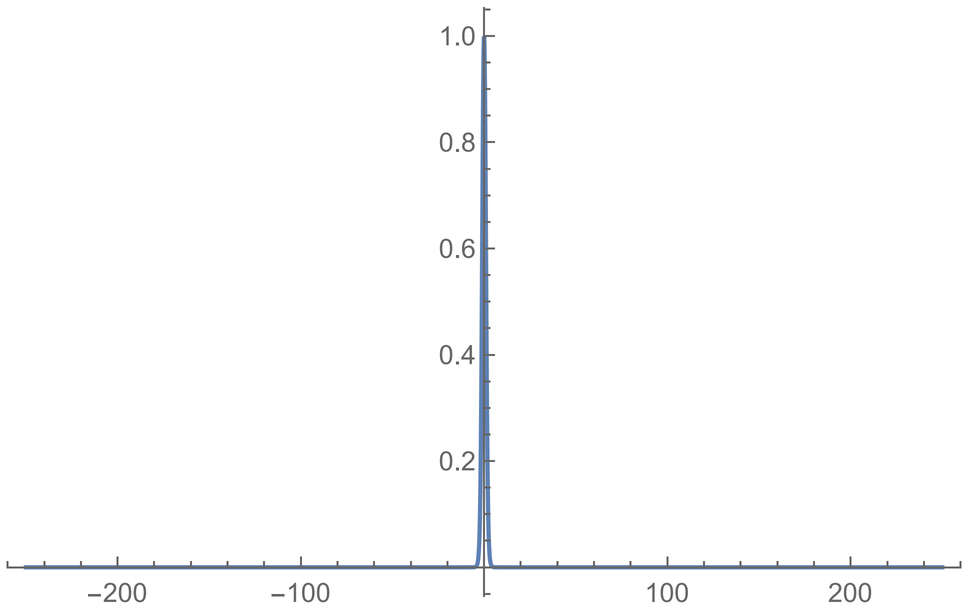

Figure 1: σ = R = 1

While it may appear to be close to a Kronecker Delta Function, a gap at the base is visible. Upon zooming in further, the curve becomes obvious as shown in Figure 2. This curve flattens to a top hat function very quickly as σ increases as illustrated in Figures 3 – 5. For a fixed field of view, the separation inherent in the top hat behavior for increasing values of R coalesces as presented in Figure 6. I found these behaviors in the mathematical models intriguing. Further function modeling will aid in understanding the equations, their implications, and their interactions. This will help advance understanding in the field. I constructed Figures 1 – 5 using Mathematica 12.0 and Figure 6 with Matlab 2019a.

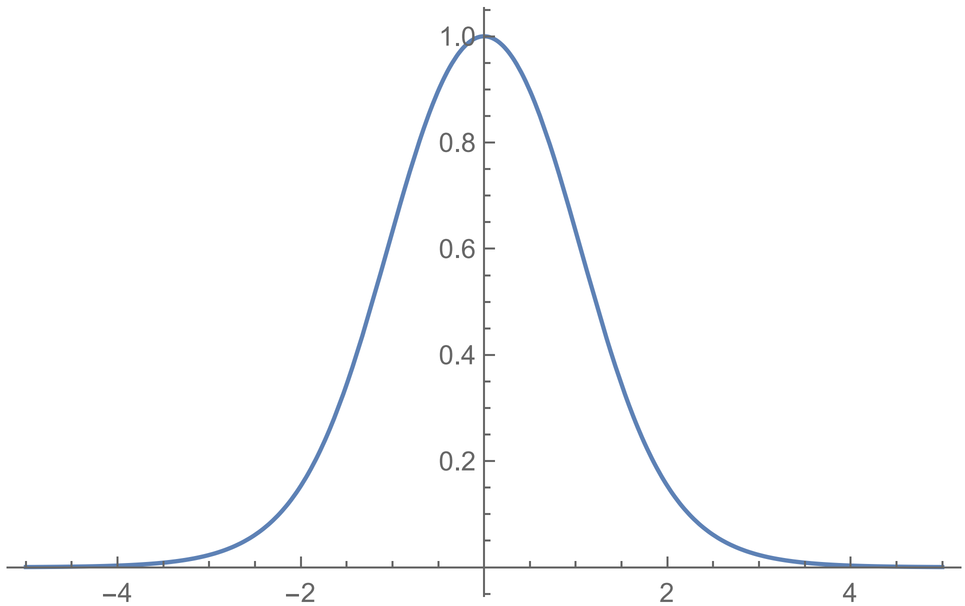

Figure 2: $\sigma = R = 1$ with a Narrower Range of r_s Values

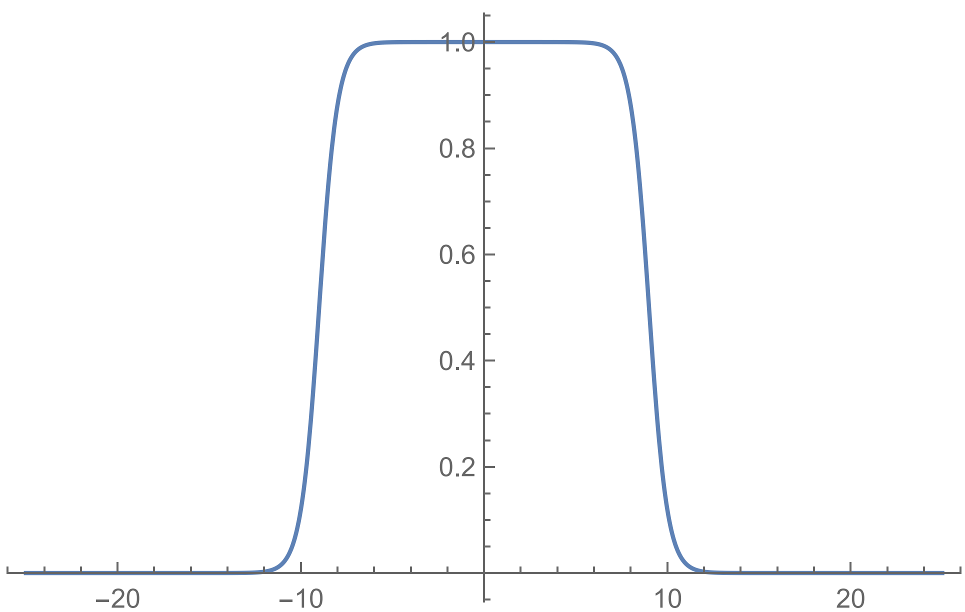

Figure 3: $\sigma = 1$, $R = 9$ Showing Flattened Top and Curving Behavior

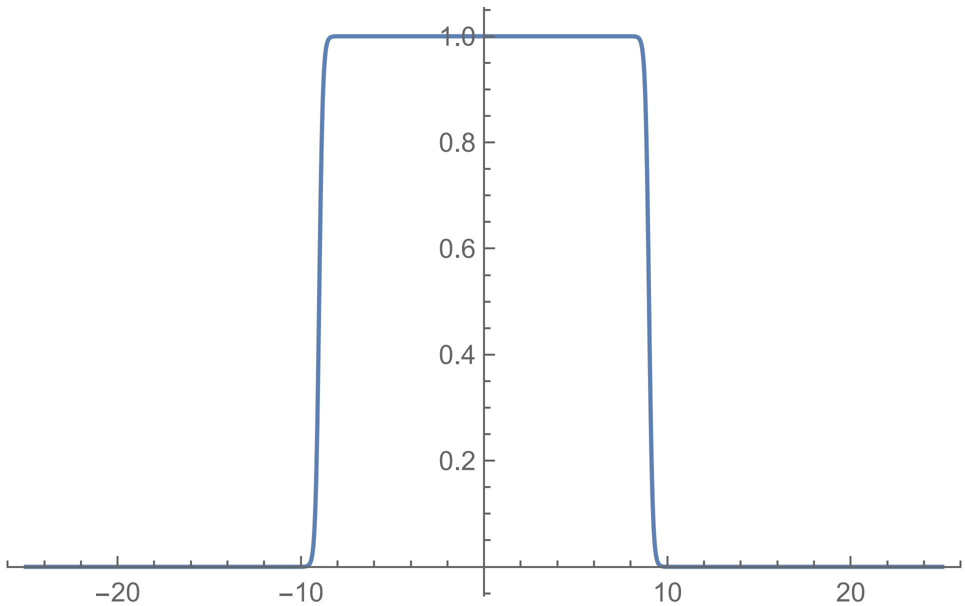

Figure 4: σ = 5, R = 9 Showing Rapid Function Straightening

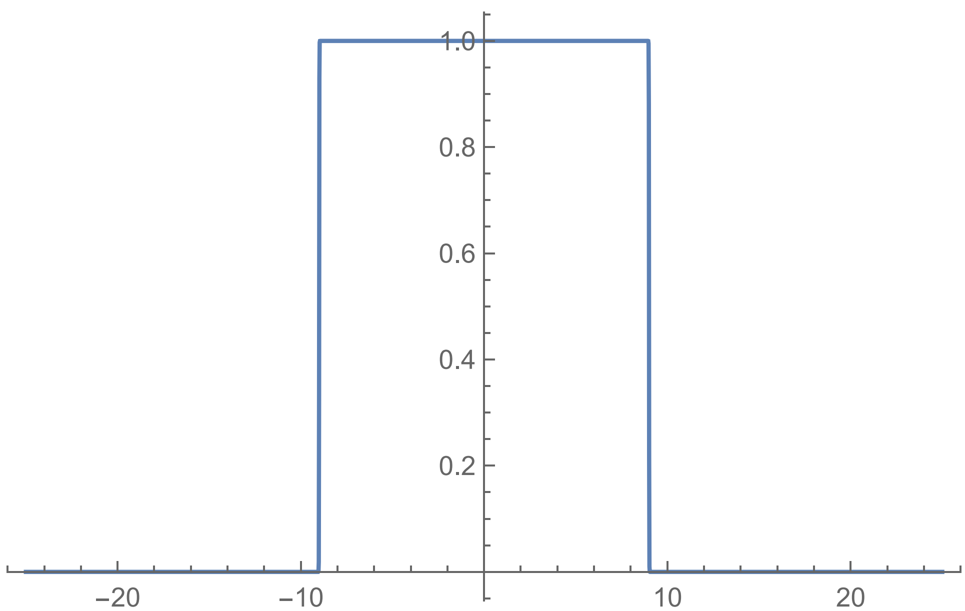

Figure 5: σ = 100, R = 9 Demonstrating Nearly Straight Edges

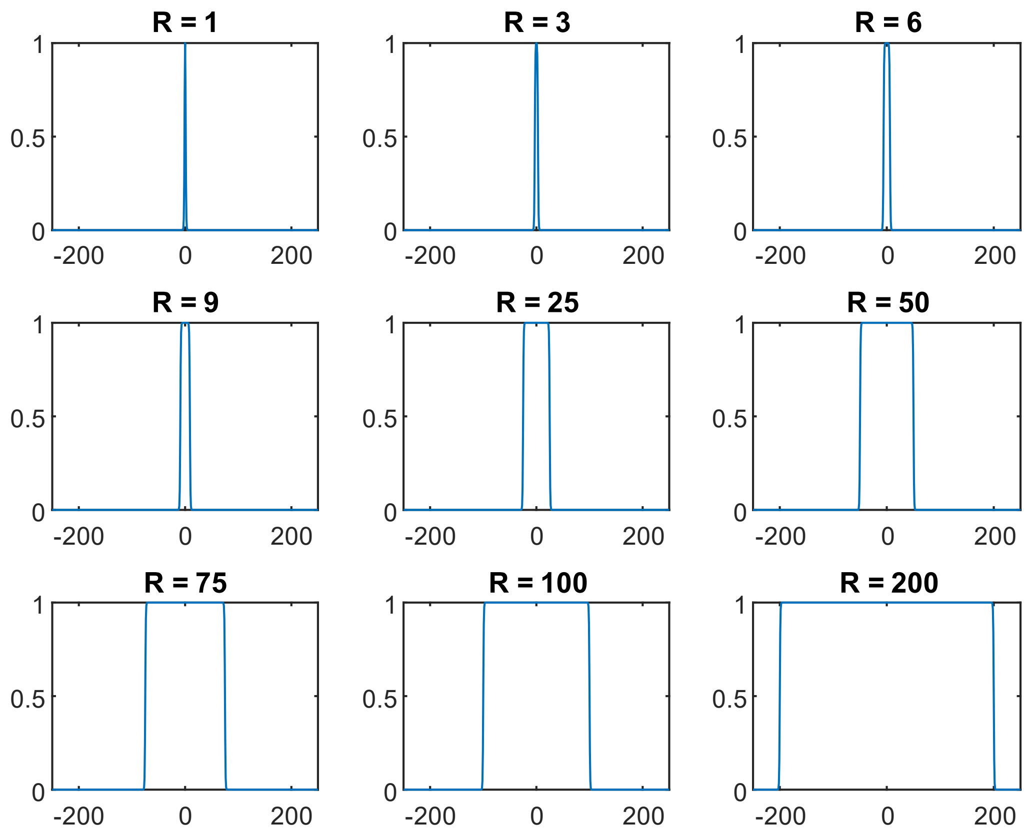

Figure 6: σ = 1, R Varies as Labeled

Current Literature

Some of the more notable recent works worth more analysis include the following. A 2018 Master's Thesis by Nicholas Driver on warp drive spacetimes which may serve as scaffolding for a future personal attempt [6]. A 2017 paper by Jusufi et. al. examining the effects of Hawking radiation on a warp bubble, which not only builds more literature, but helps evaluate the effects inside a spacetime bubble [7]. A 2017 paper by Fernando Loup examines both the Alcubierre spacetime metric and a further iteration by Natario to develop a bottle-neck shaped bubble with a large inner diameter, but effectively microscopic outer diameter with safety and navigation implications [8]. An article co-written by Lobo and Alcubierre in a 2017 publication further explaining the conceptual thinking behind warp drive, targeted at upper undergraduates and graduate students is also worth considering since it includes the original author after years of observing the literature evolve [9]. There was also a 2019 feasibility study conducted by Joseph Agnew that looked at a different swath of articles, presenting information worth considering [10]. All of these are articles I intend to more fully read in the future after improving my understanding of General Relativity and tensor mathematics so that I can more thoroughly engage in the development of this field of theoretical General Relativity.

Warp Drive Theory is still a highly theoretical field of General Relativity. These articles deal with determining the physical and mathematical methods required to explain how a spacetime bubble would work. More theoretical examination is needed before experimentation can begin. Based on existing research, small-scale experimentation may be possible after a further 50 years of modeling and mathematical development.

WHY WARP DRIVE

Warp Drive Theory interests me because I want to explore beyond the confines of our solar system. Our galaxy and universe at large have so much to offer. We have only begun to scratch the surface. We have discovered at least 55 exoplanets orbiting in the habitable zones of their stars [11]. Up until the 2015 New Horizons flyby of Pluto, we had barely resolved the planet beyond merely identifying its existence [12]. Imagine the immense advances to astronomy and astrophysics that would result from sending probes to distant star systems for close examination. Exobiology is in its infancy because of our lack of extrasolar probes. Functional warp drives would allow for incredible advances. It is mathematically likely that other species exist beyond our solar system. FTL travel would help with verifying or falsifying that assumption by collecting new data.

Further, even if no life currently exists beyond our solar system, having FTL transportation would enable our species to continue to thrive and expand onto other planets orbiting different stars, further decreasing our odds of going extinct from freak accidents of nature or berserk humans. Our planet has already experienced a few mass-extinctions. During the height of the Cold War, our species came close to destroying itself through nuclear annihilation. We have suffered the pandemics of the Black Death, the 1918 Influenza, and now COVID-19. Spreading our species throughout the galaxy will increase genetic diversity through regional adaption and distribute the possible attack centers such that we will become far more resilient than we are today.

Obtaining Results

Experimental results will first require significant theoretical breakthroughs. Some of the largest hurdles include obtaining enormous amounts of energy needed through materials not yet discovered. Some of the quoted literature mention metamaterials — which amounts to materials not yet discovered with our desired properties. Star Trek fiction utilizes dilithium crystals to contain the energy. The best existing math I found puts us in a position to need about the amount of energy contained in a few solar masses worth of exotic matter to construct a warp bubble [13]. Even assuming we could obtain that much of something we have yet to observe, putting it in a starship is absurd because of its size. However, as my colleague Shakeila Davis noted, we could use an artificially created black hole to collapse the requisite matter down to a workable size, given the increased understanding and engineering inherent in such an endeavor. Further, it could even serve as a local source of gravity, which would greatly improve the spaceship occupants' quality of life. However, this insane energy requirement could be fixed by varying the size and shape of the bubble, such that we may only need a school-bus worth of negative-energy mass-equivalence instead [14]. That possibility requires more clear research before I can comment on it.

Before any experiments can be effectively conducted, a theoretical basis must first be developed. For my contributions to be significant, I will need to deeply review existing high-profile literature, including many I mentioned earlier in this paper. However, before I can start reviewing content, I must significantly improve my understanding of the base components. I plan to begin that journey by taking introductory classes in General Relativity (I need to expand upon my existing base in Special Relativity), taking a deep dive into tensor mathematics, and reading Einstein's original publication. After building a solid framework for General Relativity in my head, I will return to all of the papers mentioned earlier, conduct mathematical modeling, and build a thesis to expound upon the base concepts and condense a further refined set of next steps for either theoretical or experimental research, depending upon what I can obtain. Part of this process will also include obtaining an M.S. in physics, focusing on General Relativity. I will eventually pursue a PhD focused on furthering this. Based on my expected career trajectory that will likely be 10 to 15 years from now.

Conclusion

This was my first review of Warp Drive literature with the base mathematical understanding necessary to understand some of the mathematics present in the literature. After examining a few equations from Dr. Alcubierre's original paper in-depth, it became clear that I should pursue graphical modeling of each equation to better conceptualize the mathematics involved. Seeing graphs morph as individual variables changed and showing the derivation of another equation were both very exciting. After strengthening my understanding of tensor mathematics and General Relativity, I will continue examining past work, contributing my analyses of existing literature, and eventually adding new insights.

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